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Motivation, Autonomy Support, and Mathematics Performance: A Structural Equation Analysis

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ABSTRACT

We develop and test a model describing the effects of motivational resources and autonomy support on math performance. The model, based on Self-Determination Theory (SDT), incorporates the assumptions that intrinsic forms of motivation positively affect math performance, whereas external regulation (another form of motivation) negatively affects math performance. We also assume that math self-concept affects math performance, not only directly but also indirectly through the mediating variable of intrinsic motivation. Finally, we examine the effects on motivation and performance of one specific aspect of teacher behavior, namely autonomy support in the classroom. We hypothesize that autonomy support can affect math performance both directly and indirectly through the mediator of math self-concept. We tested this causal model using data from the Third International Mathematics and Science Study-Revised or TIMSS-R (1999) on the math performance of eighth-grade students in the USA. The results were consistent with the predictions of SDT. Both structural equation modeling and multilevel path modeling analyses confirmed that intrinsic motivation positively influences math performance, while external regulation negatively influences math performance. Also, a positive math self-concept significantly affects math performance both directly and indirectly through the mediator of intrinsic motivation. Finally, autonomy support in the classroom significantly affects math performance both directly and indirectly through the mediator of math self-concept

INTRODUCTION

In recent years, there has been much interest regarding motivation and its effect on achievement. When students enjoy learning, they are more likely to show interest in, value, and put effort into achievement, to perform well and to persist in school (e.g., Miserandino, 1996). When students don't enjoy learning, they are less likely to show interest in and effort toward achievement, and more likely to perform poorly and drop out of school (Ryan, 1995). Self-Determination Theory (Deci & Ryan, 1985, 1991; Ryan, 1995) is one attempt to comprehensively analyze and explain the effects of motivation.

Self-Determination Theory

Self-Determination Theory or SDT (Deci & Ryan, 1985, 1991) is a theory of motivation, but combines traditional empirical methods and a theory that deals with people's internal resources for motivation (Ryan & Deci, 2000). SDT proposes that people's psychological needs are the basis for their motivation. In particular, the needs for autonomy, competence, and relatedness are believed essential for enhancing motivation. The need for autonomy is the need to engage in self-directed behavior (deCharms, 1968; Ryan & Grolnick, 1986). The need for competence (Deci & Ryan, 1985; Harter, 1978) is the need to experience satisfaction in improving one's abilities. The need for relatedness is the need to feel related to significant others. Consequently, people engage in behaviors to support these needs (Deci & Ryan, 2000; Vallerand, 1997). Research results have generally supported these claims. For example, some researchers found that students' perceived autonomy (Grolnick & Ryan, 1987), competence, and relatedness affect their motivation (Deci & Ryan, 1985). Researchers have applied SDT to various domains, such as school learning (Deci & Ryan, 1985), psychology (Ryan & Deci, 2000), physical exercise (Vallerand, 1997), and health care (Williams & Deci, 1996) among others.

Self-Regulations

SDT distinguishes between *autonomous* and *controlled regulation*. *Autonomous regulation* is *"*self-determined motivation*",* which is motivation controlled by internal forces, whereas *controlled regulation* is motivation controlled by external forces (Deci & Ryan, 1985). Motivational needs form a continuum, according to SDT, that ranges from intrinsic motivation, through extrinsic motivation, to amotivation (Deci & Ryan, 1985). At one end of this self-determination continuum is *intrinsic motivation*, which is motivation to engage in behaviors so as to experience enjoyment (Deci & Ryan, 1985). For example, an individual who says, *"I participate in <physical activity> because it is fun"*

shows intrinsic motivation. At the other end of the self-determination continuum is *amotivation*, which is defined as when individuals do not value the activity (Ryan, 1995). Amotivated individuals are neither intrinsically nor extrinsically motivated.

According to SDT, extrinsic motivation covers the continuum between amotivation and intrinsic motivation, and can be differentiated into *external regulation, introjected regulation, identified regulation*, and *integrated regulation* (Ryan & Connell, 1989; Vallerand, 1997). The least autonomous form of extrinsic motivation is *external regulation*, which is motivation to engage in behaviors controlled by rewards (Deci & Ryan, 1985). For example, an individual who says, "*I need to do well in math to get the job I want"* shows external regulation. A second type of extrinsic motivation is *introjected regulation,* which is motivation to engage in behaviors to avoid guilt, or to seek self-and other-approval (Ryan & Deci, 2000). For example, an individual who says, "*I participate in <active sports> because I will feel bad about myself if I do not"* shows introjected regulation. A more autonomous, or self-determined, form of extrinsic motivation is *regulation through identification,* which is shown when individuals value the activity (Ryan & Deci, 2000). The most autonomous form of extrinsic motivation is *integrated regulation,* which is motivation to engage in behaviors to achieve a personal goal (Deci & Ryan, 1991). For example, an individual who says, *"I go to school because I see the importance of learning"* shows identified regulation.

Vallerand's Model of Intrinsic & Extrinsic Motivation

Working in the area of sports psychology, Vallerand (1997, 2001) proposed a motivational sequence of "social factors affecting psychological factors, which affect types of motivation, which affect consequences". Social factors are the origin, mastery, and performance climate of one's social environment (Vallerand, 1997, 2001). An origin climate supports people's autonomy; mastery climates support acquiring new skills or knowledge; but performance climates support a valuing of one's ability (Dweck, 1986). The psychological factors examined by Vallerand (1997) are perceptions of autonomy, competence, and relatedness. Deci and Ryan (2000) proposed that the need for autonomy (e.g., deCharms, 1968), competence, and relatedness are essential for enhancing motivation. Consequences are the affective, cognitive, and behavioral outcomes of self-regulatory styles (Vallerand, 1997, 2001).

Needs affecting Motivation: A Self-Determination Perspective

SDT (Deci & Ryan, 1985, 1991) proposes that the need for autonomy (deCharms, 1968), competence (White, 1963), and relatedness affect motivation. When some researchers (Deci & Ryan, 1985) distinguished between autonomy-supportive versus controlling environments, they hypothesized that autonomy-supportive climates would enhance intrinsic motivation, and that controlling climates

would undermine intrinsic motivation. Much of the research guided by SDT has supported this reasoning. For example, some researchers found that when teachers or parents supported autonomy, children showed better intrinsic motivation (Deci & Ryan, 1985).

SDT (Deci & Ryan, 1985) argues that self-concept can enhance intrinsic motivation, but it will not do so, unless accompanied by autonomy. In other words, people must feel both autonomous and competent to be intrinsically motivated. Past research (Ryan, & Deci, 1985, 2000) has shown that autonomously motivated people show greater self-concept, and that autonomous motivation and self-concept both affect relevant behaviors (e.g., school performance).

Academic Achievement as a Motivational Consequence

Self-determination theory (SDT) postulates that self-determined, or autonomous, motivation is related to positive academic and emotional outcomes (including school achievement), whereas non-self determined motivation is related to negative outcomes (Deci & Ryan, 1991). Evidence exists to support this idea. For example, some researchers have found that more self-determined motivation was related to better academic performance (Miserandino, 1996), lower dropout (Vallerand et al., 1997), better ability to cope with failures, and higher quality learning (Deci & Ryan, 1991). Other researchers have found that non-self determined motivation is related to higher dropout, less interest, less value, and less effort toward achievement (Vallerand, 1997).

Motivation and Mathematics Achievement

There is little research into this theoretical framework in mathematics learning specifically. In the context of the TIMSS assessment, Mullis and colleagues (Mullis et al., 2000) found that students who show positive attitudes toward math were more likely to perform well. Ramseier (2001) used data from the TIMSS-R to develop and test a motivational model. This model, also grounded in SDT, found five types of motivational orientation among Swiss students participating in the TIMSS-R. These orientations, in decreasing degree of self-determination, are: intrinsic orientation, long-term utility orientation, achievement orientation, approval orientation, and amotivation. Having a more self-determined orientation was associated with more interest in lifelong learning and better math achievement.

Research Purpose

The major purposes of the present study were as follows: 1) to develop a model of the effects of motivational resources (e.g., intrinsic motivation and external regulation) on math achievement in a sample of US 8th graders, 2) to use data from the TIMSS-R study of math performance to test major assumptions of SDT, 3) to examine whether math self-concept explains additional and significant variance in math performance, after we control for the effect of intrinsic motivation on math performance, and 4) to examine whether autonomy support in the classroom predicts math performance both directly and indirectly through the mediator of math self-concept of student.

Our Proposed Model

We propose a model of the effects of motivation on math performance. In developing our model of motivation, we drew upon the theoretical perspectives of Deci & Ryan (1985, 1991) and Vallerand (1997). A schematic diagram summarizing our model appears in Figure 1. In the model, we propose: 1) that motivational resources affect math performance; 2) that math self-concept affects math performance both directly and indirectly through the mediator of intrinsic motivation; and 3) that autonomy support predicts math performance both directly and indirectly through the mediator of math self-concept.

====Insert Figure 1 about here====

In the present model, we add teachers' autonomy support in the classroom as a predictor of math self-concept of students. This model extends previous work (especially Vallerand, 1997) by incorporating teacher practices (autonomy support in particular) and student math self-concept, and by evaluating the effects of these factors on the criterion variable of math performance. We also add demographic control variables (gender and mother's education) as covariates. In considering demographic variables as potential covariates, we drew upon prior results by Mullis et al. (2000). Mullis et al. (2000) found that there was a significant gender difference in math self-concept and attitudes towards mathematics, and that parents' education was positively related to students' math achievement.

Hypotheses

Based on the theoretical perspective of Deci and Ryan (1985, 1987, 1991), Vallerand, Fortier, and Guay (1997), and Vallerand (1997), and consistent with past research (Grolnick & Deci, 1998; Ramseier, 2001), five hypotheses relating to motivational resources and autonomy support were incorporated as causal assumptions in the present model, as follows:

H1a: Intrinsic motivation positively affects math performance.

H1b: External regulation negatively affects math performance.

H2: Math self-concept positively affects math performance.

H3: Autonomy support in the classroom positively affects math performance.

H4: Math self-concept significantly affects math performance through the mediator of intrinsic motivation.

H5: Autonomy support in the classroom significantly affects math performance through the mediator of math self-concept.

METHOD

This study used data from the TIMSS-R 1999. An overview of the methodology used in the TIMSS-R (1999) study is presented in this section. In particular, TIMSS participants and procedures are described. Using data collected in the background questionnaires of the TIMS-R, we developed measures of autonomy support, intrinsic motivation, external regulation, introjected regulation, and math self-concept. Data analysis for the present study is described in the last section.

TIMSS Participants, Procedures, and Materials

Participants in the TIMSS-R (1999) study in the U.S. were 9,072 eighth-grade students, ranging in age from 13 to 14 years, from 150 sampled schools. The participants had an average age of 14.2 years. 50.2% were female and 49.8% were male. 90.4% were from English-speaking backgrounds.

The TIMSS-R (1999) used a two-stage sampling procedure to ensure a representative sample of U.S. students. After the researchers randomly selected a sample of schools, they randomly selected one or two math classrooms in the sampled schools. Participants were given a TIMSS 1999 student background questionnaire in the sampled classes. They were asked to respond to the student questionnaire measuring students' home backgrounds, how they spend their time out of school, their math self-concept, and their attitudes toward math (Mullis et al., 2000).

The NRCs and International Study Center staff developed the TIMSS math test, and reviewed the items and scoring guides. They also reviewed item statistics computed to determine the difficulty of each item, the reliability of the scoring of response items, and whether there were any biases (Mullis et al., 2000). Items consisted of five content areas: *fractions and number sense; measurement; data representation analysis and probability; geometry;* and *algebra.* Five performance expectations were covered in the math test: *knowing, using routine procedures, investigating and problem solving, mathematical reasoning*, and *communicating* (Mullis et al., 2000).

One-fourth of the questions in free-response format required students to write short answers. Two independent scorers who did not know the scores assigned by the others scored the free response items in one-quarter of the test booklets (Mullis et al., 2000). The inter-rater reliability between the two independent scorers was .99.

The Present Study

This section discusses the selection of a modeling subsample for the present study, and describes key variables. In order to use the TIMSS background questionnaire data to test the hypotheses of the present study, measures were developed of autonomy support, intrinsic motivation, external regulation, introjected regulation, and math self-concept.

There were eight different tests, with some questions occurring in multiple tests. Test items were grouped into 26 mutually exclusive test item "clusters". There were eight test booklets, each containing seven clusters. Math was emphasized in four booklets, with 34, 37, 37, and 38 minutes for math. Science was emphasized in four booklets, with 18, 19, 19, and 24 for math. For the present study, we used only four booklets to enhance comparability across booklets.

For the present study, we measured math performance in the TIMSS-R 1999 mathematics test with indicators such as first math overall plausible value scores or multiple imputation methods. Each student was administered only a fraction of the mathematics items. A plausible value is an estimate of how each student might have performed if they had been administered the entire set of items. Five plausible values were computed for each student, based on responses to the item set administered and using alternative models based on responses by students with similar characteristics and other items. Examining classroom performance and groups of students provide performance trends.

Autonomy Support

Autonomy support in the classroom was measured using five items from the TIMSS 1999 teacher questionnaire. In the TIMSS study, participants responded to the stem "In math lessons, how often do students_____?" The stem was followed by five items such as "work individually without assistance from the teacher" (see Table 1). Responses were indicated on a 4-point Likert scale, which ranged from 1(*never/almost never*) to 4 (*every lesson*).

Motivation

Measures were developed of intrinsic motivation, external regulation, and introjected regulation. Intrinsic motivation was measured using three items from an index of positive attitudes towards mathematics (PATM) of the TIMSS 1999 student questionnaire. An example items from PATM was: "I like math." External regulation was measured using three items from the TIMSS 1999

student questionnaire. An example item from the student questionnaire was "I need to do well in mathematics to get the job I want." Introjected regulation was measured using five items from the student questionnaire. An example item from the student questionnaire was "I need to do well in mathematics to please myself" (see Table 1). In the TIMSS study, students were asked to rate their motivation towards mathematics on a four-point Likert scale, which ranged from 1 *(strongly agree*) to 4 (*strongly disagree*).

Math Self-Concept

Math self-concept (assessment of personal characteristics regarding math ability) was measured using five items from an index of students' self-concept in mathematics (SCM) of the TIMSS 1999 student questionnaire. An example item from SCM is: "I'd like math much more if it were not so difficult" (see Table 1). In the TIMSS study, students were asked to rate their math self-concept on a four-point Likert scale, which ranged from 1 (*strongly agree*) to 4 (*strongly disagree*).

====Insert Table 1 about here====

Data Analysis

First, structural equation modeling (SEM) analysis was performed to examine 1) whether motivational resources predict math performance; 2) whether math self-concept explains additional variance in math performance, controlling for the mediating effect of intrinsic motivation on math performance; and 3) whether autonomy support predicts math performance both directly and indirectly through the mediator of math self-concept. Second, multilevel path modeling analysis was performed.

Structural Equation Modeling

Structural equation modeling (SEM) analysis was performed, using the statistical program Mplus (Muthen & Muthen, 1998). SEM is a set of statistical techniques that include confirmatory factor analysis (CFA) and path modeling (Ulman, 2001). The full model (Figure 1) features four exogenous variables (autonomy support, gender, mother's education, and external regulation) and three endogenous variables (math self-concept, intrinsic motivation, and math performance).

We organized the scores from each scale to fit the a priori conceptual models, and conducted a series of SEM analyses. Several indices were used to assess model fit: the chi-square statistic, the *comparative fit index* (CFI), the *non-normed fit index* (NNFI), the *root-mean-square error of approximation* (RMSEA), and the *standardized root mean-square residual* (SRMR) (Hu & Bentler, 1999), the Akaike information criterion (AIC), and the Bayesian information criterion (BIC).

The overall fit of the model to the data was examined using the chi-square test. A nonsignificant chi-square indicates the model to be an acceptable fit to the sample data (Bollen & Long, 1993). However, because the chi-square statistic is very sensitive to sample size (Kline, 1998), we examined a χ^2/df ratio to check fit. A ratio of 3 or less indicates a good fit for large samples (Kline, 1998).

The CFI, the NNFI, and the RMSEA were used as incremental fit indices. The CFI ranges from 0 (poor fit) to 1 (perfect fit) - cutoff values greater than .95 show acceptable fits to the data (Hu & Bentler, 1999). The NNFI ranges from 0 (poor fit) to 1 (perfect fit) - cutoff values greater than .95 show acceptable fits to the data (Hu & Bentler, 1999). RMSEAs less than .05 represent a "close fit", and values up to .08 show errors of approximation (Browne and Cudeck, 1993). The SRMR was used as an absolute fit index. The SRMR value should be .08 or less (Hu & Bentler, 1999). The AIC and the BIC were used to find parsimonious models that best fit the observed data. We can choose the model that results in the lowest value.

Multilevel Path Modeling

After we constructed a single-level SEM model, we examined the intraclass correlation for the first math plausible value. Then we tested to see whether the complete multi-level model fits the data well.

RESULTS

Descriptive Statistics & Correlation Analyses

We begin with an overview of descriptive statistics and correlation analyses for measures. Descriptive statistics for the student-level variables and class-level variables are presented in Tables 2 and 3. In the study, there are 4,566 students, 449 teachers, and 221 schools. The modal number of teachers per school is 2.03. Sample sizes average about 20.66 students per school, and 10.17 students per teacher. Bivariate correlations among study variables are presented in Table 4.

====Insert Table 2 about here====

====Insert Table 3 about here====

====Insert Table 4 about here====

Scale Reliabilities

Scale reliabilities for the measures are as follows. The number of items and the internal consistency for each scale are: autonomy support - five items (α = .71); intrinsic motivation - three items (α = .78); external regulation - three items (α = .68); introjected regulation - five items (α = .68);

and math self-concept - five items (α = .86). Most of the alpha coefficients ranged from .71 to .86, and were satisfactory on the basis of Nunnally's (1978) criterion of .70.

Structural Equation Modeling

The results reported in this section use "plausible value" scores as the criterion measure of math achievement. Plausible value score is a measure of total score, but adjusted to make different booklets comparable. The methodology for computing plausible values is described in item response theory. Structural equation modeling analysis was performed using the statistical program, Mplus (Muthen & Muthen, 1998). Two models were defined. Model 2 is related to Model 1 as follows: In Model 2, the paths between autonomy support and intrinsic motivation, between gender and intrinsic motivation, and between gender and math performance were fixed to zero, because they were non-significant in the full model (Model 1).

Fit indices for each of the two models of math achievement appear in Table 5. To begin the nested models analysis, we tested to determine whether the full model (Model 1) fits the data well. The fit of the full model (Model 1) was satisfactory: χ^2 (10) = 29.183 (i.e., under 3.00), CFI = 0.979 (i.e., greater than .95), TLI = 0.961 (i.e., greater than .95), RMSEA = 0.039 (i.e., less than .05), SRMR = 0.034 (i.e., less than .08). We tested one restricted model (Model 2 nested within Model 1) to see whether a simpler conceptualization of math achievement might provide a better fit to the data than did Model 1 (the full model). The models that were compared are listed in Table 5, along with their fit statistics.

====Insert Table 5 about here====

The model-fitting process suggested that Model 2 best fit the data (χ^2 (13) = 38.170 (i.e., under 3.00), CFI = 0.993 (i.e., greater than .95), TLI = 0.987 (i.e., greater than .95), RMSEA = 0.029 (i.e., less than .05), SRMR = 0.024 (i.e., less than .05). The model with the lowest AIC is preferred (Kline, 1998). Model 2 had the lowest AIC. On this basis, we selected Model 2 as the best model of all the models and based further analyses on that model. Model 2 is called the trimmed full model (Figure 3).

The path diagram that shows the fully-unstandardized parameter estimates for Model 2 appears in Figure 3. These findings support the model. First, the results show that intrinsic motivation positively affects math achievement (β = 7.865), whereas external regulation negatively affects math achievement (β = -15.196), consistent with the first hypothesis (*H1*). Math self-concept positively affects math achievement (β = 11.399), consistent with the second hypothesis (*H2*). Autonomy support positively affects math self-concept (β = 0.072) and math performance (β = 1.474), consistent with the third hypothesis (*H3*). Math self-concept significantly affects math performance through the mediator of intrinsic motivation, consistent with the fourth hypothesis (*H4*). Finally, a teacher's support of

autonomy in the classroom significantly affects math performance through the mediator of math self-concept, consistent with the fifth hypothesis (*H5*). The R^2 values were 0.303, and 0.218 for intrinsic motivation and math achievement respectively. Model 2 (the trimmed full model) as a whole accounted for 22% of the variance in math achievement.

====Insert Figure 2 about here====

====Insert Figure 3 about here====

Multilevel Path Modeling

As a first step, we constructed a single-level SEM model, and the model showed good fit to the data (see SEM section). Therefore, as a second step, we examined the intraclass correlation for the math first plausible value, which was .53. The intraclass correlation value of .53 shows that a substantial proportion of the variance, around 53%, is between classes. Thus, classroom-variables might account for the differences in math performance. This suggests that a multi-level analysis is desirable. Third, we tested to see whether the complete multi-level models (multi-level versions of the trimmed full model and the full model) fit the data well. The fit of the Multi-level Model 2 (multi-level version of the trimmed full model) was satisfactory: χ^2 (5) = 13.665 (i.e., under 3.00), CFI = 0.989 (i.e., greater than .95), TLI = 0.981 (i.e., greater than .95), RMSEA = 0.029 (i.e., less than .05), SRMR (B) = 0.001 (i.e., less than .08), SRMR (W) = 0.029 (i.e., less than .08), see Table 6).

The path diagram that shows the fully unstandardized parameter estimates for Multi-level Model 2 appears in Figure 5. At the within-class level, the results show that intrinsic motivation positively affects math performance (β = 2.530), whereas external regulation negatively affects math performance (β = -8.700), consistent with the first hypothesis (*H1*). Math self-concept positively affects math performance (β = 7.405), consistent with the second hypothesis (*H2*). Math self-concept significantly affects math performance through the mediator of intrinsic motivation, consistent with the fourth hypothesis (*H4*). For the between-class model, autonomy support positively affects math performance (β = 2.536), consistent with the third hypothesis (*H3a*), although it does not significantly affect math self-concept.

We can determine the variance accounted for at each level in Multi-level Model 2 (Figure 5). For the between-class model, the variables in the model accounted for 0.6% of the between-class variance in mathematics performance, with the 99.4% representing the errors in the equations, (i.e., residual variance = 0.994). The within-class variables accounted for 17% of the within-class variance in mathematics performance, with 82.8% representing the errors in the equations (i.e., residual variance = 0.828). Finally, for Multi-level Model 2 with random slopes (see Figure 7), we obtain the estimated equation predicting intrinsic motivation as follows:

Math Performance = 386.457 + 0.849(Autonomy Support) + 1.074(Intrinsic Motivation) + 0.141(Intrinsic Motivation*Autonomy Support)

We obtain the estimated equation predicting math self-concept as follows:

Math Performance = 386.457 + 0.849(Autonomy Support) + 6.690(Math Self-Concept) + 0.058(Math Self-Concept*Autonomy Support)

====Insert Table 6 about here====

====Insert Figure 4 about here====

====Insert Figure 5 about here====

====Insert Figure 6 about here====

====Insert Figure 7 about here====

DISCUSSION

According to Self-Determination Theory (SDT), students who enjoy learning are more likely to show interest and perform well. Students who don't enjoy learning are less likely to show interest and perform well (Miserandino, 1996; Deci, 1995). Perceptions of self-determination and self-concept are student motivational resources that affect their performance. Teachers can play a role in helping students develop these motivational resources by providing autonomy-supportive classrooms, which support students' needs for self-determination. Research guided by SDT (Ryan, 1995) has had a concern with these issues. Specifically, SDT proposes that self-determined motivation is related to positive academic and emotional outcomes, whereas non-self determined motivation is related to negative outcomes (Deci & Ryan, 1991). The present results confirm that intrinsic motivation positively affects math performance, whereas external regulation negatively affects math performance.

Our findings also show that a unique and substantial proportion of math achievement arises from the student's math self-concept. In other words, math self-concept explains significant variance in math performance, even after we control for the effect of intrinsic motivation on math performance. In fact, we have found that math performance is determined by motivation, but even more by math self-concept. Both predictors, intrinsic motivation and math self-concept, accounted for unique variance in math performance, and the motivational model as a whole accounted for 22% of the variance. By itself, the variable of intrinsic motivation explained 2.25% of the variance in math performance (2.25% represents the square of the r = .15 correlation between intrinsic motivation and math performance). Adding the variable of math self-concept allowed us to explain an additional 19.75% of the variance in math performance.

In offering this, we recognize the possible importance of additional aspects of self-concept and also self-efficacy. Self-efficacy expectations and outcome expectations are known to affect student performance. Bandura and his colleagues (1999) proposed that academic self-efficacy focuses on perceived ability to achieve academic outcomes, and consists of children's beliefs in their efficacy to master academic participants.

Finally, the interpretation of the present results depends on the causal assumptions we have made in developing the models. Some researchers (Marsh & Yeung, 1997) have raised questions about the causal ordering of academic self-concept and academic achievement. For example, Byrne (1984) proposed that changes in academic self-concept lead to changes in subsequent academic achievement. Some researchers (Marsh & Yeung, 1997) found that prior math self-concept significantly affected subsequent math achievement, and that prior math achievement significantly affected subsequent math self-concept. It is also possible that math self-concept is influenced by a student's ability, which affects math performance as well.

SDT (Deci & Ryan, 1985) proposed that the need for autonomy (deChrams, 1968), competence, and relatedness (Reis, 1994) are essential for achievement. Past research has supported this proposal. Consistent with the predictions of SDT, in the present study teacher's support of autonomy in the classroom positively predicted math performance. However, the relative strength of the autonomy path to math performance was weak. It may be that the effects of autonomy support by teachers vary depending on the class climate. Consistent with the predictions of SDT, math self-concept significantly predicted math performance. This suggests that math self-concept plays an important role in predicting significant math performance. In classrooms, students who have a low math self-concept are less likely to perform well.

One unexpected finding in the present study was that math self-concept was found to influence intrinsic motivation negatively. It could be that students who have a high math self-concept are more likely to think that their present math courses are boring, which leads to a low intrinsic motivation, as measured by the TIMSS questionnaire items (e.g., "Math is boring.").

SDT argues that self-perceptions of autonomy and competence should interact to increase well-being (Deci & Ryan, 1985). Past research (Grolnick, Ryan, & Deci, 1991) showed that autonomously motivated people show greater self-concept, and that autonomous motivation and self-concept both affect school performance. Consistent with the predictions of SDT, math self-concept significantly affected math performance through the mediator of intrinsic motivation. This suggests that intrinsic motivation is a significant mediator of the math self-concept-performance relationship.

Consistent with the predictions of SDT, in our "flat" SEM Models, with autonomy support treated as a level-1 variable, autonomy support significantly affected math performance both directly

and indirectly through the mediator of math self-concept, and they did so in a way that was above and beyond the effect that intrinsic motivation had on math performance. However, it is best to try to separate potential individual and class level variability among the variables included in the study. Our application of multilevel modeling techniques allowed the hierarchical examination of individual, group, and cross-level effects within a hierarchical structure. Our essential finding was that when autonomy support was appropriately treated as a level-2 variable, teachers' autonomy support in the classroom significantly affected math performance, consistent with the predictions of SDT.

Effects of relevant demographic variables

Several recent studies have reported findings from the TIMSS assessment concerning the effects on achievement of important demographic variables, including gender and parent's education level. For example, Mullis et al. (2000) found that there was a significant gender difference in math self-concept and attitudes towards mathematics internationally, but that gender was not related to math performance. Mullis et al. (2000) also found that parents' education was positively related to students' math achievement. The pattern across countries was that eighth-grade students whose parents had more education were also those who had higher achievement in math. In the present study, mother's education significantly affected math performance through the mediator of math self-concept., both in the flat SEM models and when autonomy support was approximately treated as a level-2 variable. One of the primary contribution of our study may be that a multilevel modeling analysis supported a model in which a teacher's support of autonomy in the classroom (a level-2 variable) positively affected the math achievement of students.

Limitations

Three aspects of the present research limit the generalizability of the findings. First, school contexts affect some of the relationship tested here. Deci and Ryan (1985, 1991) assumed that the degree to which the social context satisfies the need of autonomy, competence, and relatedness must affect motivation and behavioral consequences, irrespective of culture and gender. Although we considered individual and class level variability among the variables included in the study in implementing the self-determination framework (Deci & Ryan, 1985), and tested gender effects in the present study, it would have been useful to test school effects and the interaction of school and gender.

Second, we relied on data from a self-report questionnaire to assess our important theoretical variables. Several researchers such as Vallerand (1997) have previously found that self-report measures predict students' performance. However, our reliance on a self-reported questionnaire may overestimate the reliabilities that we found among the constructs.

Third, the TIMSS data was collected using a cross-sectional design. Experiences affecting one's math self-concept and intrinsic motivation can occur over time. Therefore, a longitudinal research design might more effectively estimate causal effects in our model.

Conclusions

Our findings have practical implications. SDT argues that self-perceptions of autonomy and competence should interact to increase well-being (Deci & Ryan, 1985). Past research (Grolnick, Ryan, & Deci, 1991) showed that autonomously motivated people show greater competence, and that autonomous motivation and self-perceived competence both affect school performance. Consistent with the predictions of SDT, when teachers support autonomy of students in the classroom, they provide a classroom climate that fosters math achievement. As teachers try to support students' competencies, they are more likely to foster students' self-perceptions of competence, which, in turn, promotes math achievement.

SDT (Deci & Ryan, 1985) proposes that the needs for autonomy (deChams, 1968) and competence (Harter, 1978) are essential for achievement. In the study, we found that a teacher's support of autonomy significantly affected math achievement (though the effect was relatively small). The present findings supported a model of motivation by implementing the self-determination framework. The findings provide some insight into how educators may begin to increase the content-area interest of students. In particular, the present results suggest that teachers should seek to foster an autonomy-supportive climate, not only in the interests of fostering positive math self-concept, but also because autonomy support directly affected math achievement. We must recognize that autonomy-promoting activities may be useful educational activities that directly affect math achievement by increasing subject matter knowledge.

SDT proposes that self-determined motivation is related to positive academic and emotional outcomes, whereas non-self determined motivation is related to negative outcomes (Deci & Ryan, 1991). Consistent with the predictions of SDT, we found that intrinsically motivated students were more likely to perform well. As intrinsic motivation and math self-concept are enhanced, these motivational resources in turn foster math achievement. When we looked at the contribution of the two motivational resources, we saw an interesting pattern concerning the size of their effects. When we looked at how motivational resources affected math performance, we found that the math self-concept effect ($\beta = 7.405$) was bigger than the intrinsic motivation effect ($\beta = 2.530$). Therefore, the present study suggests that both types of motivational resources significantly and uniquely affect achievement. It could be argued that achievement has deeper roots in math self-concept.

Duda and Hall (2000) argued that it is important for researchers to explore motivational models. The present study suggests one possible way to improve math performance, namely by

providing students with a learning climate that supports student autonomy. Such autonomy-promoting changes could positively affect students' math self-concept, thus directly and indirectly benefiting math performance.

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Table 1. List of Items and Scale.

Item		Scoring
Autonomy support		
BTBMLES1	"In math lessons, how often do students work individually without assistance from the teacher?"	1(Never/almost never) – 4 (every lesson)
BTBMLES2	"In math lessons, how often do students work individually with assistance from the teacher?"	1(Never/almost never) – 4 (every lesson)
BTBMLES4	"In math lessons, how often do students work together as a class with students responding to one another?"	1(Never/almost never) – 4 (every lesson)
BTBMLES5	"In math lessons, how often do students work in pairs or small groups without assistance from the teacher?"	1(Never/almost never) – 4 (every lesson)
BTBMLES6	"In math lessons, how often do students work in pairs or small groups with assistance from the teacher?"	1(Never/almost never) – 4 (every lesson)
Intrinsic motivation		
BSBMENJY	"I enjoy learning mathematics."	1(strongly agree) – 4 (strongly disagree)
BSBMLIKM	"I like mathematics."	1 (Like a lot) - 4 (Dislike a lot)
BSBMBORE (-)	"Math is boring."	1(strongly agree) – 4 (strongly disagree)
External regulation		
BSBMJOB	"I need to do well in mathematics to get the job I want."	1 (strongly agree) - 4 (strongly disagree)
BSBMWORK	" I would like a job that involved using mathematics."	1 (strongly agree) - 4 (strongly disagree)
BSBMSCHL	"I need to do well in mathematics to get into the <secondary school> I prefer."	1 (strongly agree) - 4 (strongly disagree)
Introjected regulation		
BSBMSELF	"I need to do well in mathematics to please myself."	1 (strongly agree) - 4 (strongly disagree)
BSBMPRNT	"I need to do well in mathematics to please my parents."	1 (strongly agree) - 4 (strongly disagree)
BSBMMIP2	"My mother thinks that it is important for me to do well in mathematics at school."	1 (strongly agree) - 4 (strongly disagree)
BSBMFIP2	"Most of my friends think it is important for me to do well in mathematics at school."	
BSBMSIP2	"I think it is important to do well in mathematics at school."	
Math self-concept		
BSBMMYT1	"I would like mathematics much more if it were not so difficult."	1 (strongly agree) - 4 (strongly disagree)
BSBMMYT2	"Although I do my best, mathematics is more difficult for me than for classmates."	1 (strongly agree) - 4 (strong disagree)
BSBMMYT3	"Nobody can be good in every subject, and I am just not talented in mathematics."	1 (strongly agree) - 4 (strongly disagree)
BSBMMYT4	"Sometimes when I do not understand a new topic in mathematics initially, I know I will never understand it."	1 (strongly agree) - 4 (strongly disagree)
BSBMMYT5	"Mathematics is not one of my strengths."	1 (strongly agree) - 4 (strongly disagree)
Achivement		
BSMMAT01	1st plausible value	

Table 2. Student-Level Descriptive Statistics.

Variable	M	SD	Minimum Value	Maximum Value	Skewness	Kurtosis
Intrinsic Motivation	4.33	1.66	2.00	8.00	.506	-.394
External Regulation	5.89	1.86	3.00	12.00	.412	-.114
Introjected Regulation	8.38	2.13	5.00	20.00	.875	1.445
Self-Concept	13.49	3.79	5.00	20.00	-.206	-.653
First Plausible value	495.75	85.87	216.87	813.38	.040	-.949

N = 4,566

Table 3. Classroom-Level Descriptive Statistics.

Variable	*M*	*SD*	Minimum Value	Maximum Value	Skewness	Kurtosis
Autonomy Support	11.37	2.00	5.00	20.00	1.087	3.207

N = 449

Table 4. Correlation Matrix for the Model Variables.

	1	2	3	4	5	6	7	8
1. Autonomy support	1							
2. Intrinsic motivation	.03*	1						
3. External regulation	-.01*	-.51***	1					
4. Introjected regulation	.03	.36***	.52**	1				
5. Math self concept	.04*	-.56***	-.30***	-.14***	1			
6. Gender	.03*	-.03	-.08***	.03	.10***	1		
7. Mother's education	-.06***	-.03	-.03	-.05**	.07***	.03	1	
8. 1st plausible value	.05**	.15***	-.10***	-.07***	.45***	.89***	.11***	1

Note. *p<.05, ** p<.01, ***p<.001.

Table 5. Fit Indices for Each of the Single-Level SEM Models-Autonomy Support Treated as Level-1.

Model	χ^2	df	CFI	TLI	RMSEA	SRMR	AIC	BIC
SEM Model								
Model 1	29.183	10	0.971	0.961	0.039	0.034	71416.771	71506.409
Model 2	38.170	13	0.993	0.987	0.029	0.024	71415.758	71488.589

Note. Models 2 is nested within Model 1. CFI = comparative fit index; TLI = NNFI = nonnormed fit index; RMSEA = root-mean-square error of approximation; SRMR = standardized root mean-square residual; AIC = Akaike information criterion; BIC = Bayesian information criterion; Model 1 = all free; Model 2 = three paths (AS->IM, G->IM, and G-> MP) were fixed to zero; ME = Mother's education; SC = Math self-concept; AS = Autonomy support; IM = Intrinsic motivation; G = Gender; MP = Math performance.

Table 6. Fit Indices for Each of the Multi-level Path Models.

Model	χ^2	df	CFI	TLI	RMSEA	SRMR(B)	SRMR(W)	AIC	BIC
ML1	8.054	3	0.961	0.951	0.064	0.009	0.038	79945.210	80058.467
ML2	13.665	5	0.989	0.981	0.029	0.001	0.029	79766.615	79917.624

Note. WL etc = Within-level Model; ML etc = Multi-level Model; ML Model 1 = Multi-level version of the Full Model; ML Model 2 = Multi-level version of the Trimmed Full Model.

Figure 1. Hypothesized Motivational Model of Math Performance,

with Demographic Control Variables (Gender and Mother's Education).

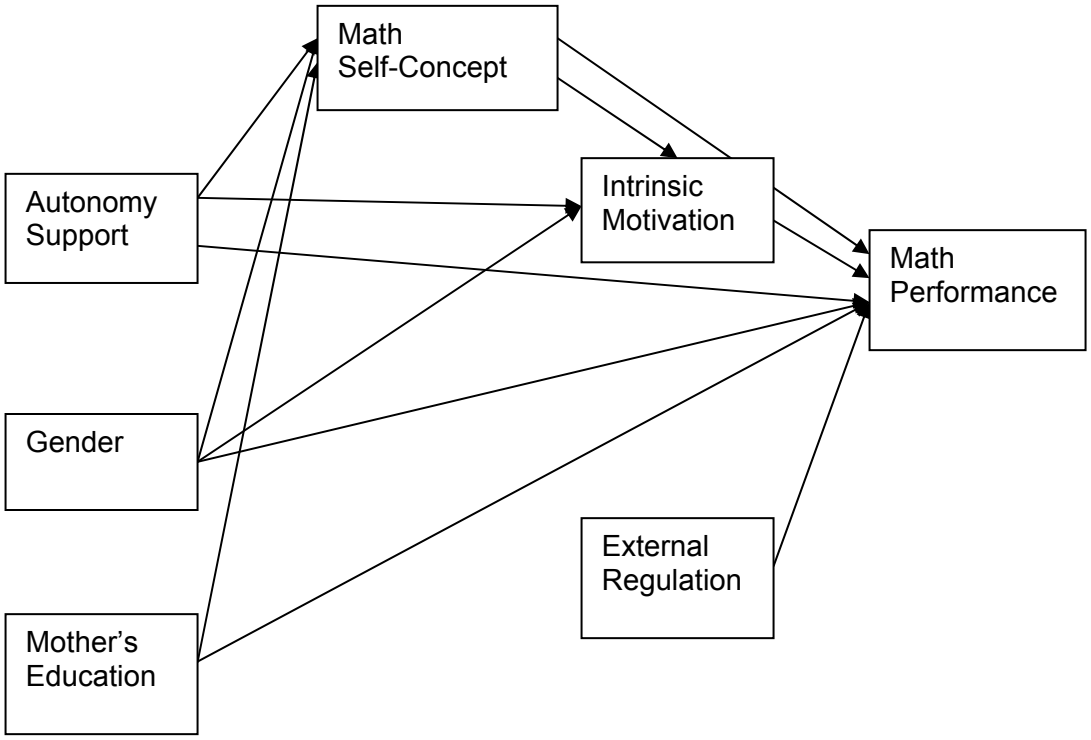
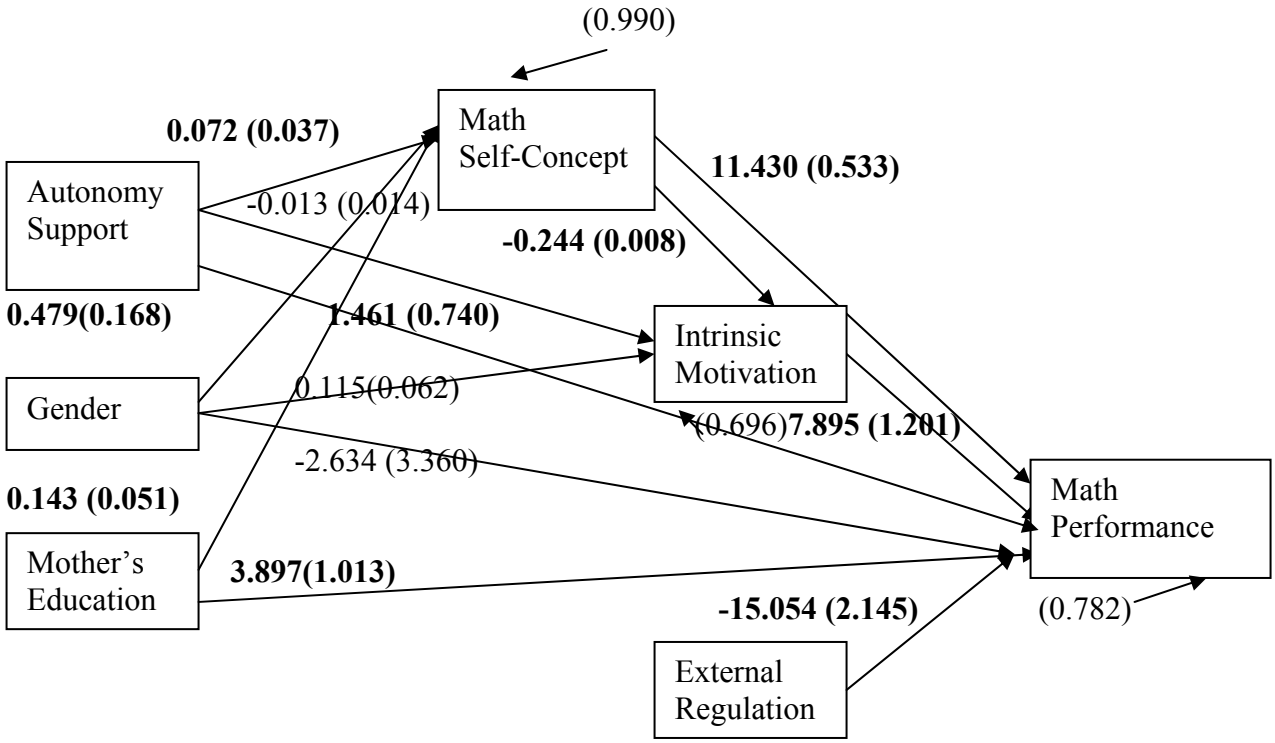
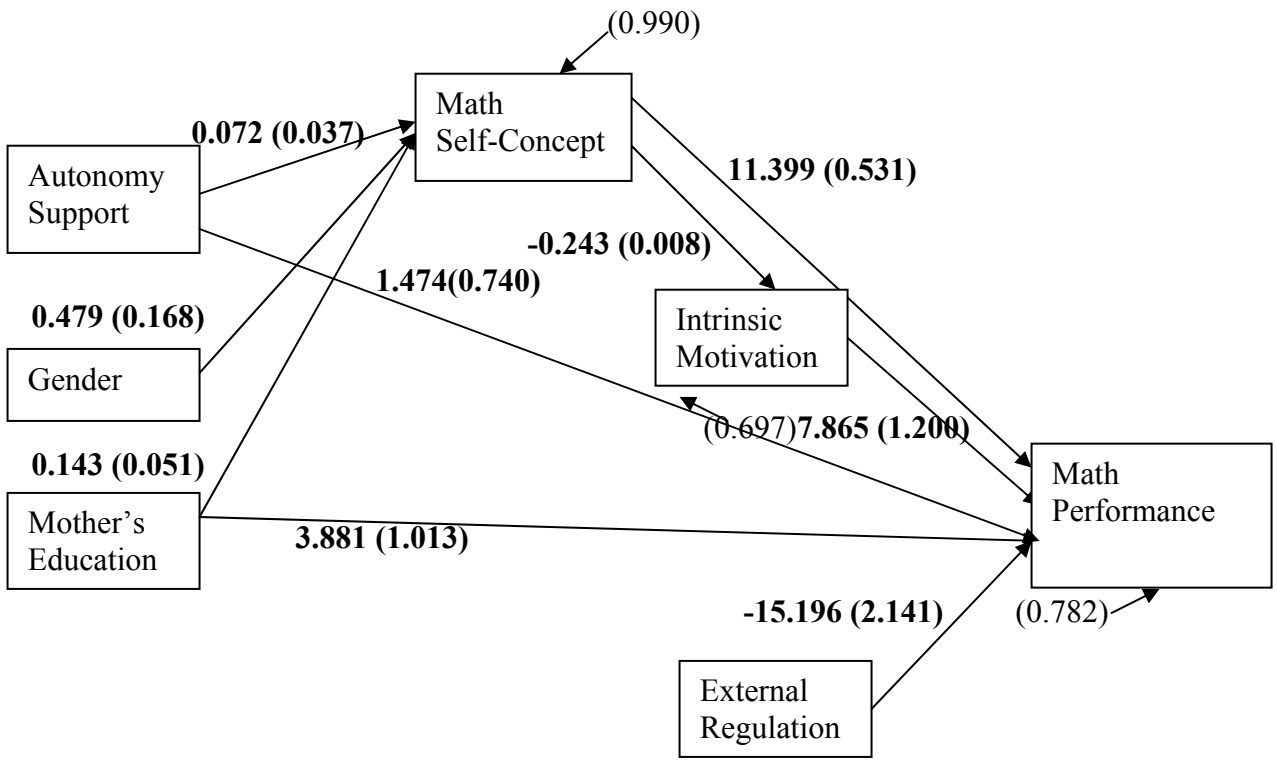


Figure 2. Results for the Full Model.



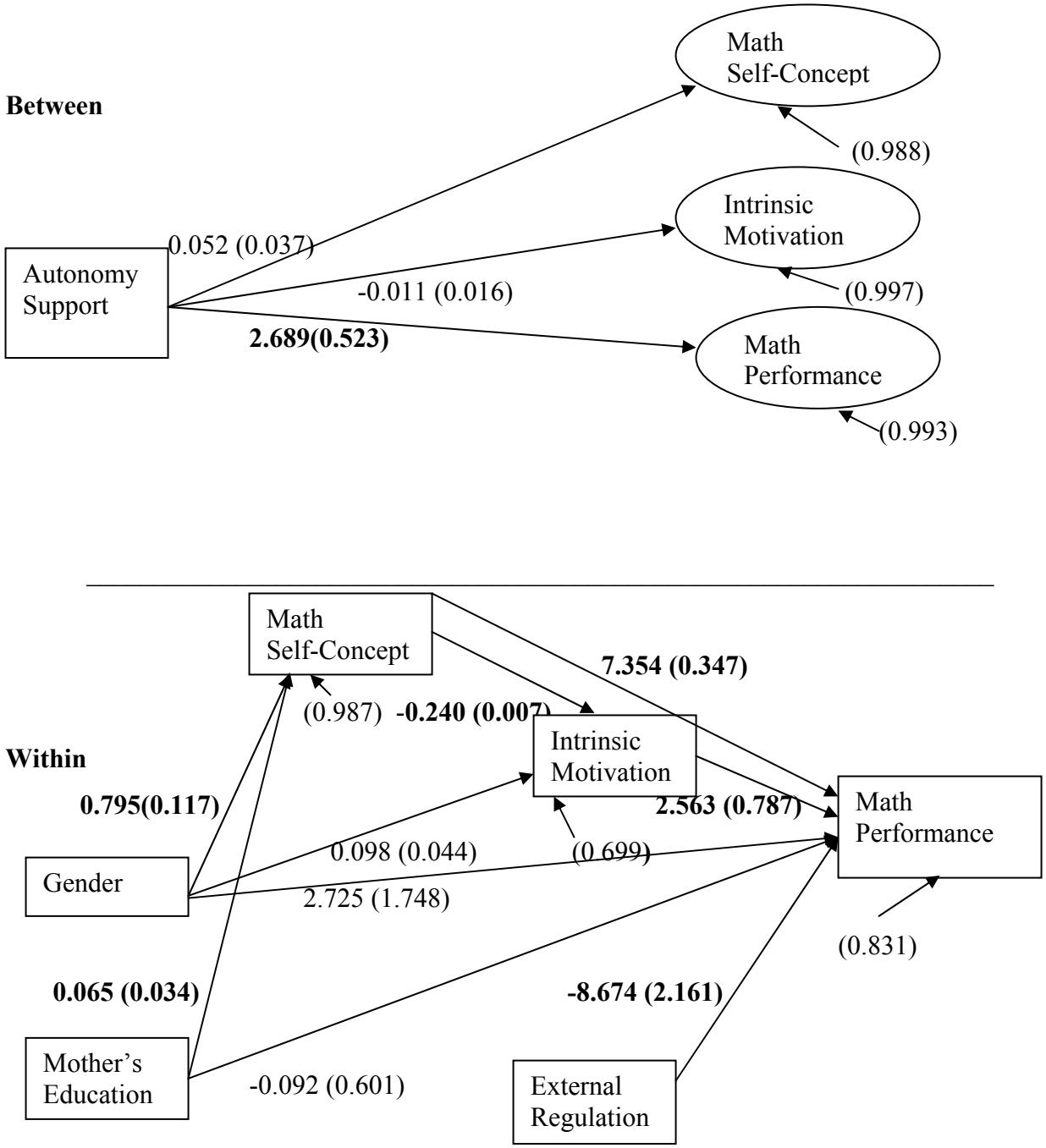
Note. Unstandardized path coefficients in boldface are significant.

Figure 3. Results for the Trimmed Full Model.



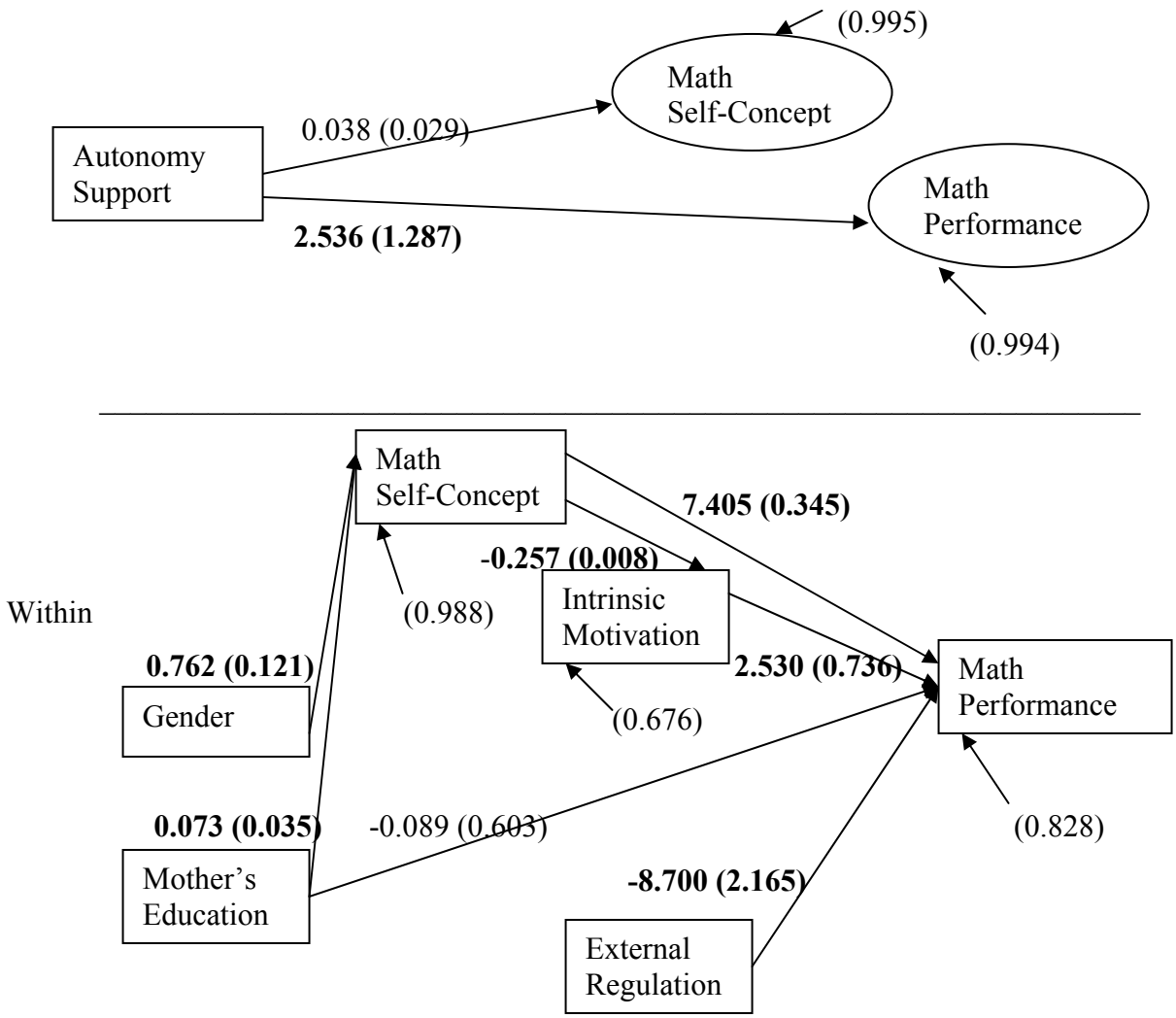
Note. Unstandardized path coefficients in boldface are significant.

Figure 4. Multi-level Model 1: Multi-level Version of the Full Model.



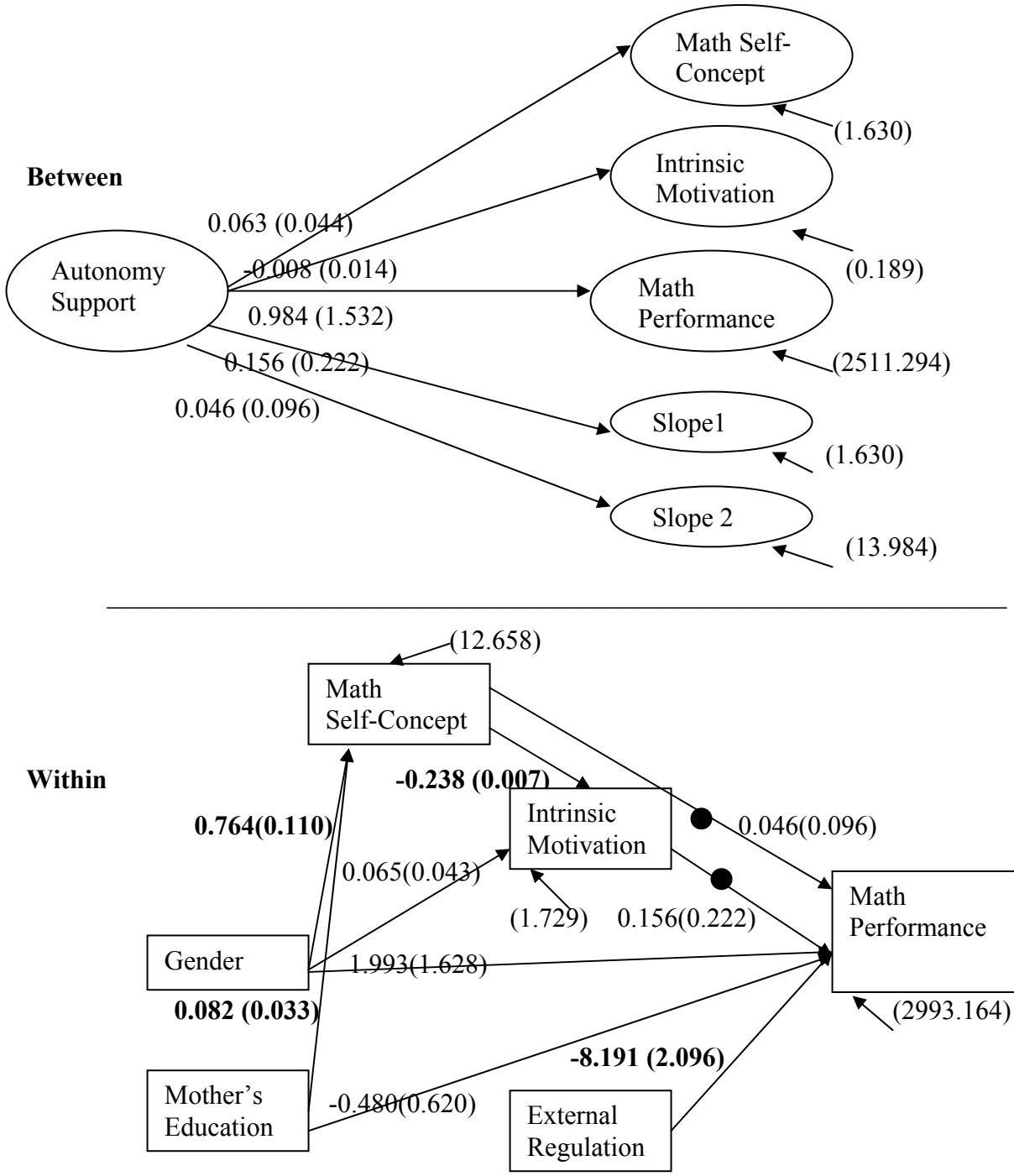
Note. Unstandardized path coefficients in boldface are significant.

Figure 5. Multi-level Model 2: Multi-level Version of the Trimmed Full Model.



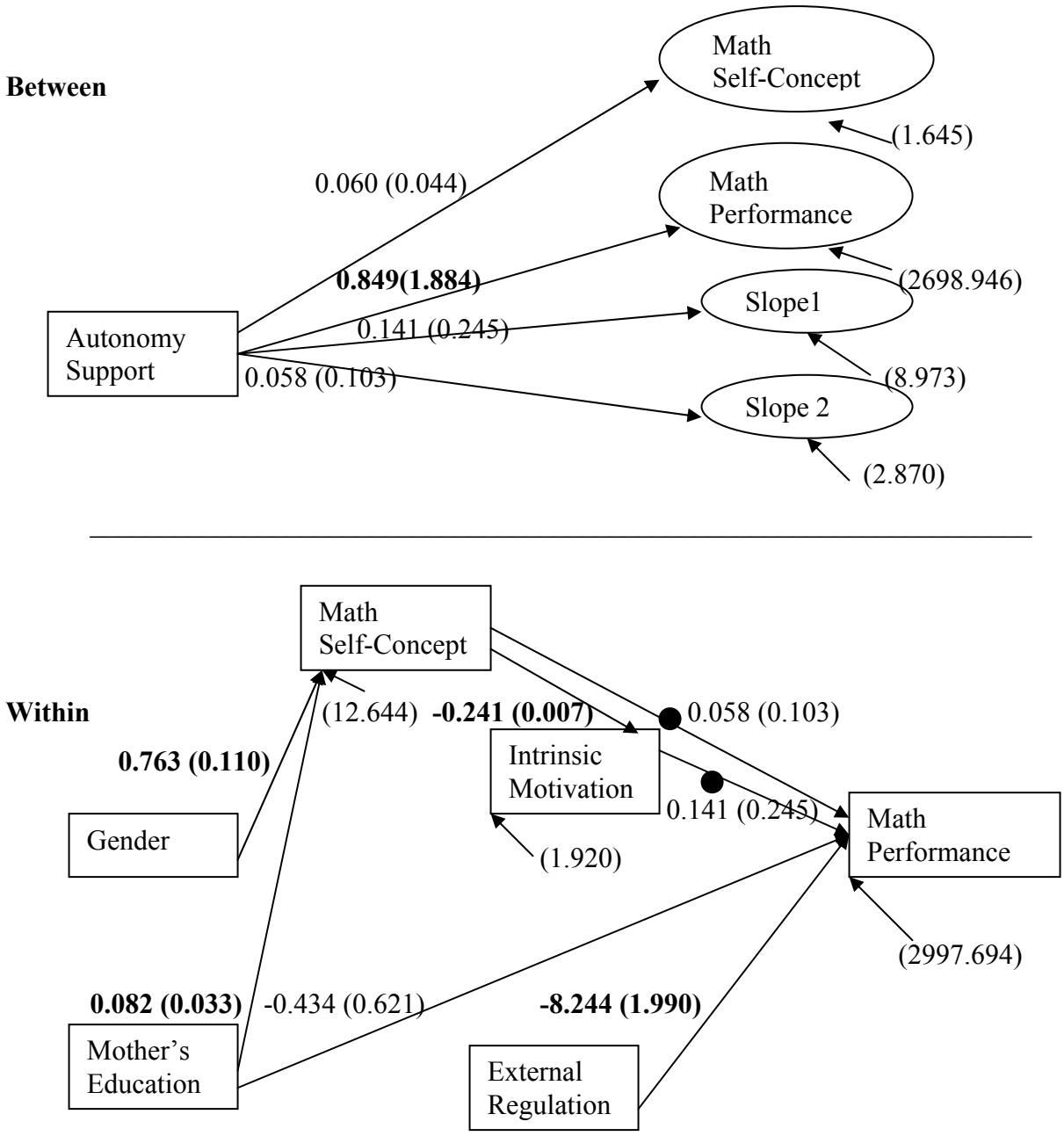
Note. Unstandardized path coefficients in boldface are significant.

Figure 6. Multi-level Model 1 with Random Slopes.



Note. Unstandardized path coefficients in boldface are significant.

Figure 7. Multi-level Model 2 with Random Slopes.



Note. Unstandardized path coefficients in boldface are significant.